BIAXIAL TENSION OF A THICK PLATE WITH AN ELLIPTICAL HOLE, FROM AGING ELASTIC-PLASTIC MATERIAL

F. B. Milyavskaya UDC 539.376

A model of an aging viscoplastic material is introduced in [1]. The yield point here is an integral operator. The development of the plastic zone under biaxial tension is considered in this paper on the basis of the model in [1] for a plate with an elliptical hole from aging material. The model of the material in the elastic-creep zone is taken according to [2, 3].

Two approximations are obtained for the stress distribution by the small parameter method [4], and the boundary of the plastic zone is determined. A numerical solution is presented for the problem. An analogous problem is considered in [4] for an ideally elastic-plastic body.

The relationships of the theory of a hereditarily aging plastic body, the equilibrium equations, the incompressibility and isotropy conditions, and the hereditary plasticity condition, for plane strain have the form [1, 4]

$$\frac{\partial\sigma_x(t)}{\partial x}+\frac{\partial\tau_{xy}(t)}{\partial y}=0,\quad \frac{\partial\tau_{xy}(t)}{\partial x}+\frac{\partial\sigma_y(t)}{\partial y}=0; \tag{0.1}$$

$$\varepsilon_x+\varepsilon_y=0,\quad \frac{\varepsilon_x-\varepsilon_y}{\sigma_x(t)-\sigma_y(t)}=\frac{\varepsilon_{xy}}{\tau_{xy}(t)}; \tag{0.2}$$

$$\frac{1}{4}\left(\frac{\sigma_x(t)-\sigma_y(t)}{k(t)}+\int\limits_{t_0}^{t}(\sigma_x(\tau)-\sigma_y(\tau))K^*(\tau,t)\,d\tau\right)^2+\left(\frac{\tau_{xy}(t)}{k(t)}+\int\limits_{t_0}^{t}\tau_{xy}(\tau)K^*(\tau,t)\,d\tau\right)^2=1. \tag{0.3}$$

Here $\sigma_x(t)$, $\sigma_y(t)$, $\tau_{xy}(t)$ are the stress components dependent on the time, ε_x, ε_y, ε_{xy} are the plastic strain rate components, $k(t)$ is the variable yield point in time, and $K^*(\tau, t)$ is the kernel of the hereditary operator.

1. Let us consider an infinite plane with an elliptical hole with the semi-axes $a(1 + c)$, $a(1 - c)$ extended by mutually perpendicular forces $p_1(t)$ and $p_2(t)$ at infinity, and let the normal pressure $p_0(t)$ act on the hole outline. We will set

$$c=d_1\delta,\ (p_1(t)-p_2(t))/2=d_2\delta, \tag{1.1}$$

where δ, d_1, d_2 are constants taking on the following values in the limits:

$$0\leqslant\delta\leqslant 1,\ 0\leqslant d_i\leqslant 1\ (i=1,\ 2). \tag{1.2}$$

For $d_1 = 0$, $d_2 = 1$ biaxial tension of a thick plate with a circular hole is evidently observed [5], while for $d_1 = 1$, $d_2 = 0$ we have a plate with an elliptical hole under normal pressure.

Let us go over to dimensionless parameters and variables while retaining the previous notation. The yield point as $t \to \infty$ is denoted by k_∞ and all the quantities with the dimensionality of a stress are referred to k_∞, and those with the dimensionality of a length to $\rho_s^{(0)}(t)$ (the radius of the plastic zone in the zeroth approximation).

The equation of the ellipse in rectangular Cartesian coordinates in the notation accepted has the form

$$x^2/[a^2(1+c)^2]+y^2/[a^2(1-c)^2]=1. \tag{1.3}$$

Let $\rho_* = \rho_*(\theta)$ be the equation of the hole in polar coordinates. Going over to the polar coordinates $x = \rho_*\cos\theta$, $y = \rho_*\sin\theta$, we convert (1.3) into

Cheboksary. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 169-174, January-February, 1989. Original article submitted September 14, 1987.

0021-8944/89/3001-0163$12.50

$$\rho_*(\theta)=\frac{a\left(1-(\delta d_1)^2\right)}{\sqrt{1-2\delta d_1\cos 2\theta+(\delta d_1)^2}}. \tag{1.4}$$

We write two approximations in the parameter δ for $\rho_*(\theta)$ from (1.4)

$$\rho_*(0)=a+ad_1\delta\cos 2\theta-\frac{3}{4}a\delta^2d_1^2(1-\cos 4\theta)+\ldots \tag{1.5}$$

We assume that the inner outline is enclosed by the plastic zone. We determine the stress components in the plastic and creep zones and the radius of the plastic zone $\rho_s^{(I)}(t)$ in a first approximation

$$\sigma_\rho^{(I)p}+\frac{\partial\sigma_\rho^{(0)p}}{\partial\rho}\rho_*^{(I)}=\frac{\partial p_\nu}{\partial\rho}\rho_*^{(I)},\quad \tau_{\rho\theta}^{(I)p}-\left(\sigma_\theta^{(0)p}-\sigma_\rho^{(0)p}\right)\dot{R}_1=\frac{\partial p_\tau}{\partial\rho}\rho_*^{(I)}, \tag{1.6}$$

where $\sigma_\rho^{(n)}$, $\sigma_\theta^{(n)}$, $\tau_{\rho\theta}^{(n)}$ are the stress components and n is the number of the **approximation;**

$$R_1=\frac{\rho_*^{(I)}}{\rho_*^{(0)}},\quad \dot{R}_1=\frac{\partial\rho_*^{(I)}}{\partial\theta} \tag{1.7}$$

($\rho_*^{(0)}=a$, $\rho_*^{(I)}=ad_1\cos 2\theta$ according to (1.5)).

The right sides in (1.6) vanish for the problem under consideration since $p_\nu=p_0(t)$ is independent of ρ while $p_\tau=0$. The expressions for $\sigma_\rho{}^{(0)}$ and $\sigma_\theta{}^{(0)}$ are obtained in [5]

$$\begin{gathered}\sigma_\rho^{(0)p}=2k(t)\varphi(t)\ln\frac{\rho}{a}-p_0(t),\quad \tau_{\rho\theta}^{(0)p}=0,\\ \sigma_\theta^{(0)p}=2k(t)\varphi(t)\left(\ln\frac{\rho}{a}+1\right)-p_0(t).\end{gathered} \tag{1.8}$$

Here and henceforth the superscript p is taken to denote the plastic zone and e the elastic-creep zone while $\varphi(t)$ is a function of the time [5].

For the first approximation the condition (0.3) has the form [5]

$$\sigma_\rho^{(I)p}=\sigma_\theta^{(I)p}=0. \tag{1.9}$$

Therefore, the boundary conditions (1.6), with (1.7)-(1.9) taken into account, are the following at the boundary $\rho_*=a$

$$\begin{gathered}\sigma_\rho^{(I)p}=-2d_1k(t)\varphi(t)\cos 2\theta,\quad \sigma_\theta^{(I)p}=\sigma_\rho^{(I)p},\\ \tau_{\rho\theta}^{(I)p}=-4d_1k(t)\varphi(t)\sin 2\theta.\end{gathered} \tag{1.10}$$

Let us write the equation (0.1) in the polar coordinate system

$$\frac{\partial\sigma_\rho}{\partial\rho}+\frac{\sigma_\rho-\sigma_\theta}{\rho}+\frac{1}{\rho}\frac{\partial\tau_{\rho\theta}}{\partial\theta}=0,\quad \frac{1}{\rho}\frac{\partial\sigma_\theta}{\partial\theta}+\frac{\partial\tau_{\rho\theta}}{\partial\rho}+\frac{2\tau_{\rho\theta}}{\rho}=0. \tag{1.11}$$

Solving the differential equations (1.11) and taking account of (1.9) and (1.10), we obtain expressions for the stress in the plastic zone in a first approximation

$$\begin{gathered}\sigma_\rho^{(I)p}=\frac{2ad_1}{\rho}\varphi(t)k(t)(\sqrt{3}\sin\chi-\cos\chi)\cos 2\theta,\quad \sigma_\theta^{(I)p}=\sigma_\rho^{(I)p},\\ \tau_{\rho\theta}^{(I)p}=\frac{4ad_1}{\rho}\varphi(t)k(t)\cos\chi\sin 2\theta\quad\left(\chi=\sqrt{3}\ln\frac{\rho}{a}\right).\end{gathered} \tag{1.12}$$

Furthermore, let us consider the creep zone under boundary conditions at infinity in the polar coordinate system

$$\begin{gathered}\sigma_\rho^{(\infty)e}=p(t)-\delta d_2\cos 2\theta,\quad \sigma_\theta^{(\infty)e}=p(t)+\delta d_2\cos 2\theta,\\ \tau_{\rho\theta}^{(\infty)e}=\delta d_2\sin 2\theta,\quad p(t)=\frac{1}{2}(p_1(t)+p_2(t)).\end{gathered} \tag{1.13}$$

From (1.13) for the first approximation

$$\sigma_{\rho(\infty)}^{(I)e}=-d_2\cos 2\theta,\quad \sigma_{\theta(\infty)}^{(I)e}=d_2\cos 2\theta,\quad \tau_{\rho\theta(\infty)}^{(I)e}=d_2\sin 2\theta, \tag{1.14}$$

and for the succeeding approximations

$$\sigma_{\theta(\infty)}^{(n)e} = \sigma_{\rho(\infty)}^{(n)e} = 0, \quad \tau_{\rho\theta(\infty)}^{(n)e} = 0 \quad (n \geqslant 2). \tag{1.15}$$

The stress distribution and the plastic zone boundary in the zeroth approximation correspond to the axisymmetric state of a plane with a circular hole [5]

$$\sigma_\rho^{(0)e} = p(t) - \frac{\varphi(t)k(t)}{\rho}, \quad \sigma_\theta^{(0)e} = p(t) + \frac{\varphi(t)k(t)}{\rho},$$
$$\tau_{\rho\theta}^{(0)e} = 0, \quad \rho_s^{(0)}(t) = \exp\gamma, \quad \gamma = \frac{1}{2}\left(\frac{p(t)}{\varphi(t)k(t)} - 1\right). \tag{1.16}$$

Let us write down the conjugate condition for solutions for $\sigma_\rho{}^{(n)}$ on the boundary $\rho = 1$:

$$\left[\sigma_\rho^{(I)} + \frac{\partial\sigma_\rho^{(0)}}{\partial\rho}\rho_s^{(I)}\right] = 0 \quad (n = \mathrm{I}),$$
$$\left[\sigma_\rho^{(II)} + \frac{\partial\sigma_\rho^{(I)}}{\partial\rho}\rho_s^{(I)} + \frac{\partial^2\sigma_\rho^{(0)}}{\partial\rho^2}\frac{(\rho_s^{I})^2}{2} + \frac{\partial\sigma_\rho^{(0)}}{\partial\rho}\rho_s^{(II)}\right] = 0 \quad (n = \mathrm{II}). \tag{1.17}$$

The conjugate conditions for the components $\sigma_\theta^{(n)}$ and $\tau_{\rho\theta}^{(n)}$ have an analogous form [4]. Taking account of (1.8) and (1.16), we obtain boundary conditions for $\rho = 1$ from the conditions (1.17):

$$\sigma_\theta^{(I)e} = 2ad_1\varphi(t)k(t)(\sqrt{3}\sin\chi^* - \cos\chi^*)\cos 2\theta + 4\varphi(t)k(t)\rho_s^{(I)},$$
$$\tau_{\rho\theta}^{(I)e} = -4ad_1\varphi(t)k(t)\sin 2\theta \quad \left(\chi^* = \sqrt{3}\ln\frac{\rho_s^{(0)}(t)}{a}\right). \tag{1.18}$$

The solution of the problem agrees identically with the solution for an elastic body [2] for an incompressible elastic-creep material. According to [6], by taking account of the boundary conditions (1.14) and (1.18) we find the stress distribution and radius of the plastic zone in a first approximation:

$$\sigma_\rho^{(I)e} = \cos 2\theta\left[\left(\frac{4}{\rho^2} - \frac{3}{\rho^4} - 1\right)d_2 + 2\sqrt{3}\,ad_1\varphi k\sin\chi^*\left(\frac{2}{\rho^2} - \frac{1}{\rho^4}\right) + 2ad_1\varphi k\cos\chi^*\left(\frac{2}{\rho^2} - \frac{3}{\rho^4}\right)\right],$$
$$\sigma_\theta^{(I)e} = \cos 2\theta\left[\left(1 + \frac{3}{\rho^4}\right)d_2 + \frac{2ad_1\varphi k}{\rho^4}(\sqrt{3}\sin\chi^* + 3\cos\chi^*)\right],$$
$$\tau_{\rho\theta}^{(I)e} = \sin 2\theta\left[\left(1 + \frac{2}{\rho^2} - \frac{3}{\rho^4}\right)d_2 + ad_1\varphi k\cos\chi^*\left(\frac{6}{\rho^2} - \frac{10}{\rho^4}\right) + 2\sqrt{3}\,ad_1\varphi k\sin\chi^*\left(\frac{1}{\rho^2} - \frac{1}{\rho^4}\right)\right],$$
$$\rho_s^{(I)} = \left(\frac{d_2\rho_s^{(0)}(t)}{\varphi(t)k(t)} + 2d_1\cos\chi^*\right)\cos 2\theta. \tag{1.19}$$

2. The equilibrium equations in the polar coordinate system have the form (1.11). Let us write the plasticity condition (0.3) in a second approximation [5] by taking the lower limit of integration $t_0 = 0$:

$$\frac{\sigma_\rho^{(II)p}(t) - \sigma_\theta^{(II)p}(t)}{k(t)} + \int_0^t 2K^*(\tau, t)\left(\sigma_\rho^{(II)p}(\tau) - \sigma_\theta^{(II)p}(\tau)\right)d\tau - \left(\frac{\tau_{\rho\theta}^{(I)p}(t)}{k(t)} + \int_0^t K^*(\tau, t)\tau_{\rho\theta}^{(I)p}(\tau)d\tau\right)^2 = 0. \tag{2.1}$$

According to the linearized boundary conditions in a second approximation [4], and by taking (1.7)-(1.9) and (1.12) into account, we obtain the boundary conditions on the boundary $\rho = a$

$$\sigma_\rho^{(II)p} = d_1\varphi k(2 - 9\cos 4\theta), \quad \tau_{\rho\theta}^{(II)p} = -6d_1^2\varphi k\sin 4\theta. \tag{2.2}$$

Taking account of (1.12) the plasticity condition (2.1) has the form

$$f_1(t, \rho, \theta) + \int_0^t K_1(\tau, t)f_1(\tau, \rho, \theta)d\tau = F^*(\rho, \theta)F(t), \tag{2.3}$$

where

$$f_1(t, \rho, \theta) = \frac{\sigma_\rho^{(II)p}(t, \rho, \theta) - \sigma_\theta^{(II)p}(t, \rho, \theta)}{k(t)};$$

$$K_1(\tau, t) = 2K^*(\tau, t)k(\tau); \quad F^*(\rho, \theta) = \left(\frac{4ad_1}{\rho}\cos\chi\sin 2\theta\right)^2;$$

$$F(t) = \left(\varphi(t) + \int_0^t K^*(\tau, t)\varphi(\tau)\,d\tau\right)^2.$$

Equation (2.3) is a Volterra integral equation of the second kind (ρ and θ are parameters), its inversion is [7]

$$f_1(t, \rho, \theta) = F^*(\rho, \theta)F(t) - \lambda\int_0^t F^*(\rho, \theta)\Gamma(t, \tau, \lambda)F(\tau)\,d\tau \tag{2.4}$$

$\left(\Gamma(t, \tau, \lambda) = \sum_{\nu=0}^{\infty}\lambda^\nu K_{\nu+1}\right.$ is the resolvent kernel and $\left.\lambda = -1\right)$. We satisfy the equilibrium equations by setting

$$\sigma_\rho^{(II)p} = \frac{1}{\rho}\frac{\partial\Phi^{(II)}}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi^{(II)}}{\partial\theta^2},$$

$$\sigma_\theta^{(II)p} = \frac{\partial^2\Phi^{(II)}}{\partial\rho^2}, \quad \tau_{\rho\theta}^{(II)p} = -\frac{1}{\rho}\frac{\partial^2\Phi^{(II)}}{\partial\rho\,\partial\theta} + \frac{1}{\rho^2}\frac{\partial^2\Phi^{(II)}}{\partial\theta^2}. \tag{2.5}$$

Substituting (2.5) into (2.3) and taking account of (2.4) we find a differential equation to determine the stress function $\Phi^{(II)}$:

$$\rho^3\frac{\partial^2\Phi^{(II)}}{\partial\rho^2} - \rho\frac{\partial^2\Phi^{(II)}}{\partial\rho^2} - \frac{\partial^2\Phi^{(II)}}{\partial\theta^2} = -k(t)f_1(t, \rho, \theta). \tag{2.6}$$

Solving (2.6) with the boundary conditions (2.2), we have the stress components in the plastic zone in a second approximation from (2.5):

$$\sigma_\rho^{(II)p} = d_1^2k\left(2\varphi - \frac{5}{2}\psi\right) + \frac{ad_1^2k}{\rho}\cos 4\theta\left[\left(\frac{19}{2}\psi - 9\varphi\right)\cos\gamma - \right.$$

$$\left. - \sqrt{15}\left(\frac{3}{2}\psi - \varphi\right)\sin\gamma\right] + \frac{a^2d_1^2k\psi}{2\rho^2}\left[(4 - \sqrt{3}\sin 2\chi + \cos 2\chi) - \cos 4\theta\,(8 + 11\cos 2\chi - 7\sqrt{3}\sin 2\chi)\right],$$

$$\sigma_\theta^{(II)p} = d_1^2k\left(2\varphi - \frac{5}{2}\psi\right) + \frac{ad_1^2k}{\rho}\cos 4\theta\left[\left(\frac{19}{2}\psi - 9\varphi\right)\cos\gamma - \right. \tag{2.7}$$

$$\left. - \sqrt{15}\left(\frac{3}{2}\psi - \varphi\right)\sin\gamma\right] - \frac{ad_1^2k\psi}{2\rho^2}(4 + 7\cos 2\chi + \sqrt{3}\sin 2\chi) - \frac{a^2d_1^2k\psi}{2\rho^2}\cos 4\theta\,(3\cos 2\chi - 7\sqrt{3}\sin 2\chi),$$

$$\tau_{\rho\theta}^{(II)p} = \frac{2ad_1^2k}{\rho}\sin 4\theta\left[(4\psi - 3\varphi)\cos\gamma + \sqrt{15}(\psi - \varphi)\sin\gamma\right] - \frac{a^2d_1^2k\psi}{\rho^2}\sin 4\theta\,(1 + 7\cos 2\chi + \sqrt{3}\sin 2\chi).$$

Here $\psi(t)$ is the solution of the integral equation

$$\psi(t) = F(t) - \lambda\int_0^t K_1(t, \tau)\psi(\tau)\,d\tau; \quad k = k(t); \quad \varphi = \varphi(t); \quad \gamma = \sqrt{15}\ln(\rho/a).$$

Taking account of (1.8), (1.12), (1.19) and (2.7) we obtain boundary conditions for $\rho = 1$ from the conjugate conditions (1.17) (second formula)

$$\sigma_\rho^{(II)e} = A + B\cos 4\theta, \quad \tau_{\rho\theta}^{(II)e} = D\sin 4\theta, \tag{2.8}$$

where

$$A = d_1^2k\left[\left(2\varphi - \frac{5}{2}\psi\right) + \frac{a^2\psi}{2}(4 - \sqrt{3}\sin 2\chi^* + \cos 2\psi^*)\right] - k\varphi\left(\frac{d_2}{k\varphi} + 2ad_1\cos\chi^*\right)^2;$$

$$B = ad_1^2k\left[\left(\frac{19}{2}\psi - 9\varphi\right)\cos\gamma^* - \sqrt{15}\left(\frac{3}{2}\psi - \varphi\right)\sin\gamma^*\right] -$$

$$- \frac{1}{2}\psi a^2(8 + 11\cos 2\chi^* - 7\sqrt{3}\sin 2\chi^*) - k\varphi\left(\frac{d_2}{k\varphi} + 2ad_1\cos\chi^*\right)^2;$$

$$D = 2ad_1^2k\left[(4\psi - 3\varphi)\cos\gamma^* + \sqrt{15}(\psi - \varphi)\sin\gamma^*\right] - a^2d_1^2k\psi(1 + 7\cos 2\chi^* +$$

$$+ \sqrt{3}\sin 2\chi^*) - 4\left(\frac{d_2}{k\varphi} + 2ad_1\cos\chi^*\right)(d_2 + 3ad_1k\varphi\cos\chi^*);$$

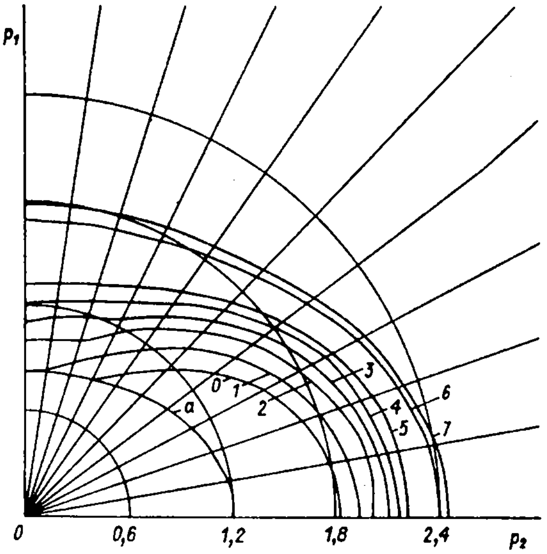

Fig. 1

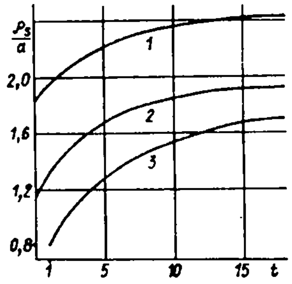

Fig. 2

$$\gamma^* = \sqrt{15}\ln\frac{\rho_s^{(0)}(t)}{a}.$$

According to [6], we find the stress components from conditions (2.8) and then the radius of the plastic zone in a second approximation

$$\begin{gathered}
\sigma_\rho^{(II)e} = \frac{A}{\rho^2} + \cos 4\theta\left[B\left(\frac{3}{\rho^4} - \frac{2}{\rho^6}\right) + 3D\left(\frac{1}{\rho^6} - \frac{1}{\rho^4}\right)\right],\\
\sigma_\theta^{(II)e} = -\frac{A}{\rho^2} + \cos 4\theta\left[B\left(\frac{2}{\rho^6} - \frac{1}{\rho^4}\right) + D\left(\frac{1}{\rho^4} - \frac{3}{\rho^6}\right)\right],\\
\tau_{\rho\theta}^{(II)e} = \sin 4\theta\left[2B\left(\frac{1}{\rho^4} - \frac{1}{\rho^6}\right) + D\left(\frac{3}{\rho^6} - \frac{2}{\rho^2}\right)\right],\\
\rho_s^{(II)} = \frac{1}{4k\varphi}(M\cos 4\theta + N).
\end{gathered} \tag{2.9}$$

Here

$$\begin{gathered}
M = \frac{a^2 d_1^2 k}{2}\left[(17\psi + 20\varphi)\cos 2\chi^* + (11\psi - 20\varphi)\sqrt{3}\sin 2\chi^* + 4(5\varphi - \psi)\right] -\\
- 4ad_1^2 k\left[(4\psi - 3\varphi)\cos\gamma^* + \sqrt{15}(\psi - \varphi)\sin\gamma^*\right] + \frac{d_2}{k\varphi}(3d_2 + 16ad_1 k\varphi\cos\chi^* -\\
- 4\sqrt{3}\,ad_1 k\varphi\sin\chi^*);\\
N = d_1^2 k(5\psi - 4\varphi) - a^2 d_1^2 k(10\varphi - 3\psi)\cos 2\chi^* - \sqrt{3}a^2 d_1^2 k(4\varphi - \psi)\sin 2\chi^* -\\
- 10a^2 d_1^2 k\varphi - \frac{3d_2^2}{k\varphi} - 4ad_1 d_2(4\cos\chi^* - \sqrt{3}\sin\chi^*).
\end{gathered}$$

3. We write the expression for the radius of the plastic zone from (1.16), (1.19) and (2.9)

$$\rho_s(t) = \exp\gamma + \delta\left(\frac{d_2}{k\varphi}\exp\gamma + 2d_1\cos\chi^*\right)\cos 2\theta + \frac{\delta^2}{4k\varphi}(M\cos 4\theta + N). \tag{3.1}$$

Let

$$\begin{gathered}
k(t) = \frac{1 - \beta\exp(-\alpha t)}{1 - \beta},\quad K^*(\tau) = \frac{\alpha\beta(1 - \beta)\exp(-\alpha\tau)}{(1 - \beta\exp(-\alpha\tau))^2},\\
\varphi(t) = \frac{(1 - \beta\exp(-\alpha t))^2}{(1 - \beta)^2},\quad p_0(t) = 0,\\
p_1(t) = 5 - 3\exp(-0.1t),\ p_2(t) = 4.6 - 3\exp(-0.1t),\\
\delta = 0.2,\ \alpha = 0.1,\ \beta = 0.2,\ d_1 = d_2 = 1.
\end{gathered}$$

Analysis of the expression (3.1) shows that no relaxation occurs if $\gamma(t)$ is a growing function. The hole outline (curve a) and the plastic zone boundary are displayed in Fig. 1 for different times, the lines 0-6 are the boundary location at the times with unit intervals starting from the time of load inclusion, and 7 as $t \to \infty$. The change $\rho_s(t)$ as a function of time is given in Fig. 2 for three directions ($\theta = 0$, $\theta = \pi/4$, $\theta = \pi/2$ are the lines 1-3), the time changes from the time of load inclusion to infinity.

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